Chapter

6

Mathematics, Science and Technology

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Science and technology have left their mark on our way of life and our environment and are among the most revealing examples of human ingenuity. Scientific discoveries and technological achievements affect major aspects of our existence. Computers, for example, have revolutionized the way we work and communicate, and even the way we think, and have become the principal tool for acquiring knowledge in many fields.

Moreover, science and technology would not have reached their current level of development without the contribution of mathematics. Although each subject area followed its own course of development, they have become more closely related as they have evolved. More often than not, technical objects with any measure of sophistication work by making use of components that operate according to the principles of mathematical logic. However, widespread use of mathematics has not been limited to the fields of science and technology. Countless situations require us to decode numerical information, estimate, calculate and measure, all of which are operations that belong to the world of mathematics.

Mathematics, science and technology each develop according to their own dynamic, but this development is also a function of the external pressure exerted by a society seeking ways to meet some of its needs. Mathematical developments, scientific discoveries and technological achievements must be placed in their historical, social, economic and cultural context if we are to understand how these three areas have evolved.

For the most part, scientific and technological advances contribute to our individual and collective well-being, but some of these advances may also threaten the ecological balance of our environment or introduce new elements whose long-term environmental effects are difficult to foresee. Only by acquiring a broad general knowledge of science and technology will students be able to take a critical look at these changes and appreciate the ethical issues they raise.

General Objective in Mathematics, Science and Technology

To provide access to a specific set of knowledges related to the methods, conceptual fields and languages specific to each of the subjects in this subject area.

Core Learnings in Mathematics, Science and Technology

- Understands information and conveys it clearly using language appropriate to mathematics, science and technology: terminology, graphics, notation, symbols and codes
- Uses inductive and deductive reasoning
- Establishes connections between the learnings he/she acquires in each subject in this particular subject area and the learnings related to other subjects
- Views this knowledge as a tool that can be used in everyday life
- Analyzes data resulting from observations or found in a problem and uses appropriate strategies to achieve a result or to find a solution that can then be explained, verified, interpreted and generalized
- Appreciates the importance of mathematics, science and technology in human history
- Exercises critical judgment in assessing the impact of mathematics, science and technology on individuals, society and the environment

Introduction

Mastery of mathematics is a significant asset when it comes to carving out a place for oneself in a society where its practical applications are as numerous as they are varied.

Mathematics is a major source of intellectual development and a determining factor in students' educational success. Its mastery is also a significant asset when it comes to carving out a place for oneself in a society where the practical applications of mathematics are as numerous as they are varied. High technology, engineering and computer programming are among the many fields requiring the use of mathematics, but it is also used in manufacturing common everyday objects, in measuring time or in organizing space.

Mathematics involves abstraction. Although it is always to the teacher's advantage to refer to real-world objects and situations, he/she must nevertheless set out to examine, in the abstract, relationships between the objects or between the elements of a given situation. For example, a triangular object becomes a geometric figure, and therefore a subject of interest to mathematicians, as soon as we begin to study the relationships between its sides, its vertices and its angles, for example.

The program is organized around three competencies: the first refers to the ability to solve situational problems; the second pertains to mathematical reasoning, which implies familiarity with concepts and processes specific to mathematics; and the third focuses on communication using mathematical language.

Mathematical activities always involve the examination of situational problems. The process of solving situational problems is a topic in and of itself, but problem solving is also an instructional tool that can be used in the vast majority of mathematical learning processes. It is of particular importance because the cognitive activity associated with mathematics involves logical reasoning applied to situational problems.

Reasoning in mathematics consists in establishing relationships, combining them and using them to perform a variety of operations in order to create new concepts and take one's mathematical thinking to a higher level. In elementary school, students develop the ability to engage in deductive, inductive and creative mathematical reasoning. They become familiar with deductive reasoning when learning how to draw a conclusion from the information given in a situational problem. They become familiar with inductive reasoning when asked to derive rules or laws on the basis of their observations. They become familiar with creative reasoning because they must devise combinations of operations in order to find different solutions to a situational problem.

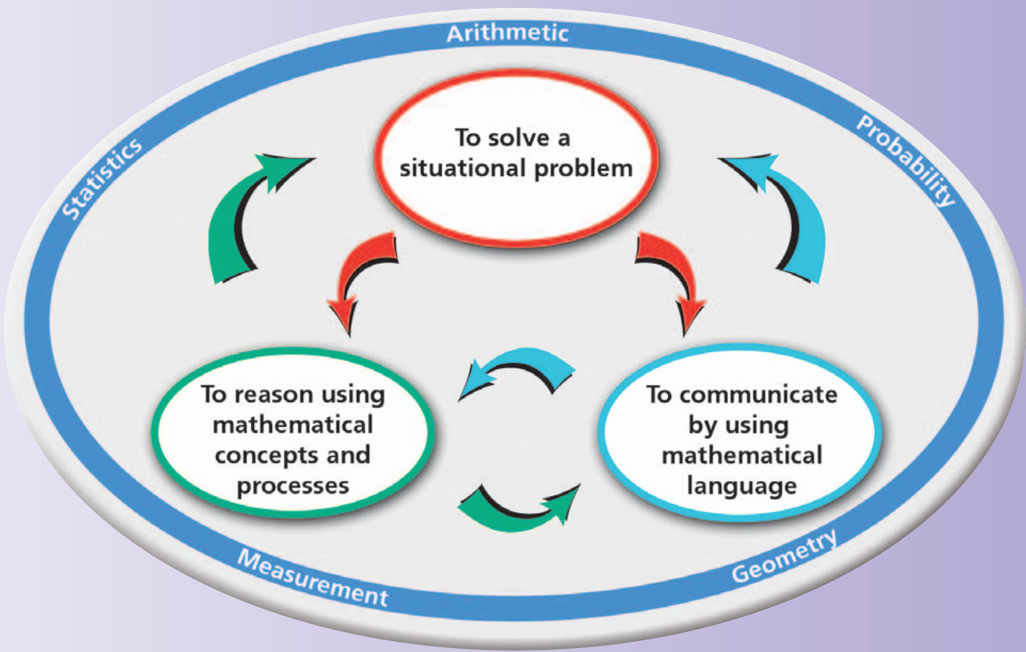
Communication using mathematical language serves two purposes: to familiarize the students with mathematical terminology and to teach them about the justification process. In the first case, the students discover new words and new meanings for known words. In the second case, they learn how to provide complete and precise explanations for a procedure or a line of reasoning.

On a different level, incorporating a historical dimension into the mathematics curriculum is an excellent way of enhancing its cultural component. This provides students with the opportunity to understand the evolution, meaning and usefulness of mathematics and to discover that the invention and development of certain instruments such as the ruler, the abacus, the protractor and the calculator were directly or indirectly related to practical needs that emerged in different societies. An overview of history can also illustrate the fact that mathematical knowledge results from the extensive work of mathematicians with a passion for their subject.

Lastly, technology can prove to be a valuable tool that will help the students solve situational problems, understand concepts and processes and carry out assigned tasks more efficiently.

The development of the three competencies covered in the program is closely connected with the acquisition of knowledges related to arithmetic, geometry, measurement, statistics and probability. These branches of mathematics include the mathematical concepts and processes studied in this program. In essence, the three competencies are distinguished by the emphasis placed on different facets of mathematical thinking, which are, in fact, all integrated. Such a distinction should make it easier to understand this thinking and to structure the pedagogical process, but it in no way suggests that these elements should be examined separately. Logically speaking, we cannot reason using mathematical concepts and processes without communicating in mathematical language, and we usually engage in mathematical reasoning when solving situational problems.

Figure 8
Mathematics



Competency 1 • To solve a situational problem related to mathematics

Focus of the Competency

Meaning of the Competency

The capacity to solve a situational problem is an intellectual process used in a wide variety of situations. On a practical level, it is spontaneously used to meet various everyday challenges. On a more abstract level, it can prove to be a powerful intellectual tool that develops reasoning and creative intuition. It can be as useful to those who wish to understand or resolve theoretical and conceptual enigmas as it is to a statistician whose work has immediate practical consequences. Relatively speaking, it is also useful to a student who is asked to find a way of determining the number of objects in a collection or calculating the area of a rectangle.

When solving situational problems in preschool and elementary school, students are engaged in a process that involves using different strategies related to comprehension, organization, problem solving, validation and communication. These problems also provide an opportunity to employ mathematical reasoning and to communicate using mathematical language.

Connections to Cross-Curricular Competencies

Because of its scope, the competency of solving a situational problem makes it possible to develop all the cross-curricular competencies. In particular, it requires students to use creativity and encourages them to process information, find efficient ways of working (often in teams) and develop appropriate ways of communicating. In all these respects, there is considerable overlap between this competency and the cross-curricular competency dealing with the ability to solve problems.

Context for Learning

A situational problem is characterized by the need to attain a goal, carry out a task or find a solution. This objective cannot be instantly achieved, since it is not an exercise involving applications. On the contrary, its achievement requires reasoning, research and the use of strategies to mobilize learnings. When solving situational problems in mathematics, the students must also perform a series of operations in order to decode, model, verify, explain and validate. This is a dynamic process that involves anticipating results, redoing certain steps in the problem-solving procedure and exercising critical judgment.

A situational problem relates to a specific context and provides a challenge geared to the students' capabilities. It must arouse and engage their interest and encourage them to take action in order to work out a solution. Lastly, it must involve some concern for metacognitive reflection.

Situational problems can require the use of arithmetic, geometry, measurement, statistics and probability. They may deal with purely mathematical questions or practical questions that are in some way familiar to the students and that are related to actual or realistic situations. Depending on the purpose of these problems, they may involve dealing with complete, superfluous, implicit or missing information.

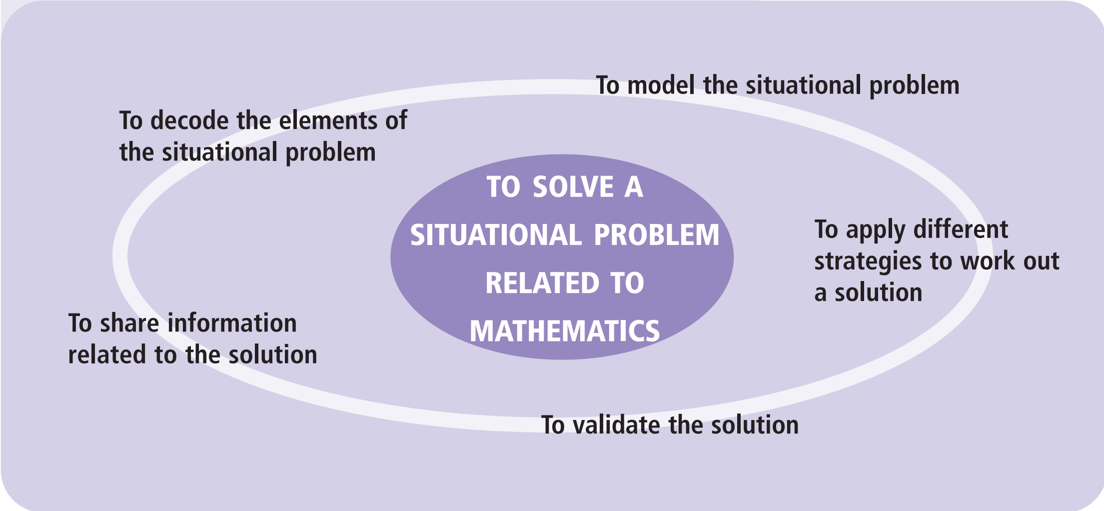
Developmental Profile

During Cycle One, the students learn how to identify the relevant information in a situational problem. They see how the information given in the situational problem relates to the assigned task. They also learn how to model a situational problem, apply different strategies and rectify their solution in light of their results and discussions with their classmates.

During Cycle Two, the students succeed in identifying the implicit information in situational problems and increase their ability to develop models and apply a variety of strategies. They can describe the procedure they have used and explain how they went about their work, and they may be interested in approaches that differ from their own.

During Cycle Three, the students are able to decode situational problems that involve recognizing situations in which information is missing. They work more independently in developing models and find it easier to devise strategies. They are better at validating their solution and commenting on their classmates' solutions.

Key Features of the Competency



Evaluation Criteria

- Production of a correct solution (procedure and final answer) ❶ ❷ ❸
- Oral or written explanation of the main aspects of the solution ❶ ❷ ❸
- Appropriate oral or written explanation of how the solution was validated ❷ ❸

Legend:* ❶ Cycle One ❷ Cycle Two ❸ Cycle Three

* This legend also applies to the Evaluation Criteria for the other competencies and to the sections entitled Essential Knowledges and Suggestions for Using Information and Communications Technologies.

End-of-Cycle Outcomes

Cycle One

By the end of this cycle, the students solve a situational problem based on complete information. They determine the task to be performed and find the relevant information by using different types of representations such as objects, drawings, tables, graphs, symbols or words. They work out a solution involving one or two steps and occasionally check the result. Using basic mathematical language, they explain their solution (procedure and final answer) orally or in writing.

Cycle Two

By the end of this cycle, the students solve a situational problem that may involve more than one type of information. They are more careful in choosing the types of representations they will use to highlight the relevant information in the situational problem, and they may also use diagrams. They anticipate the result and work out a solution involving a few steps. They validate the solution (procedure and final answer) and explain it orally or in writing using elaborate mathematical language.

Cycle Three

By the end of this cycle, the students solve a situational problem involving different types of information. They make more appropriate use of the various types of representations that allow them to organize this information. They anticipate the result, work out a solution that may involve several steps, and associate the presentation of the problem with that of similar problems. They validate the solution (procedure and final answer) and explain it orally or in writing using exact mathematical language.

COMPETENCY 2 • TO REASON USING MATHEMATICAL CONCEPTS AND PROCESSES

Focus of the Competency

MEANING OF THE COMPETENCY

To reason is to logically organize a series of facts, ideas or concepts in order to arrive at a conclusion that should be more reliable than one resulting from an impression or intuition. This does not mean that there is no place for intuition and creativity, but these faculties must be channelled into efforts that ultimately lead to a clearly stated conclusion based on a certain line of reasoning.

In mathematics, organizing means engaging in mental activities such as abstracting, coordinating, differentiating, integrating, constructing and structuring. These activities, which deal with the relationships between objects or between their components, should, for example, help the students understand the additive and multiplicative properties of a number or its ordinal and cardinal dimensions. These activities can also help them discover the meaning of iteration as it pertains to measurements, of equality or inequality in an equation and of direct or inverse proportionality.

Mathematical reasoning involves apprehending the situation, mobilizing relevant concepts and processes and making connections. In so doing, the students become familiar with mathematical language, construct the meaning of mathematical concepts and processes and establish links between them. This approach also encourages the students to use mathematical instruments.

Different examples illustrate how this competency is used. In arithmetic, the students construct the meaning of numbers, number systems and operations. In geometry, they discover the characteristics of plane figures and solids and establish spatial relationships. With regard to measurement, they study what measurements, units of measure and their interrelationships mean. With regard to probability, they examine random events by, for example, formulating their conclusions in terms of whether these events are certainties, possibilities or impossibilities. In statistics, they interpret and draw graphs representing various aspects of everyday life.

With respect to processes, the students spontaneously devise their own ways of doing things by using instruments or technology, and explore these methods in order to understand how they work. For example, instead of using recognized algorithms, the students can begin by carrying out arithmetic operations based on relatively unstructured intuition. Measurements can be taken using any object as a unit of measurement. However, mathematics has its own processes and instruments that have become well-established conventions over time. In addition, when it comes to learning about instruments, the instructional goal should be to ensure that the students are able to use these conventional tools intelligently and to understand what they are doing, while developing their measurement sense.

CONNECTIONS TO CROSS-CURRICULAR COMPETENCIES

When the students use mathematical concepts and processes, they also develop cross-curricular competencies, especially intellectual competencies relating to the exercise of critical judgment and the use of creativity. They also use the methodological competency relating to the development of effective work methods, as well as the communication-related competency.

CONTEXT FOR LEARNING

Fostering the development of this competency involves using situational problems that will force students to ask themselves questions, to make connections between the elements they are examining and to look for answers to their questions. These situations deal with arithmetic, geometry, measurement, statistics and probability and occasionally relate to the history of mathematics.

The students primarily use manipulative materials, make use of technology and consult a resource person, if necessary. They use tools ranging from an ordinary piece of graph paper to a computer. When they use processes that call for specific instruments (e.g. ruler, protractor, balance, calculator), they are encouraged to study the development of measuring systems and instruments or computational processes. They are asked to make a list of mathematical processes and tools used in everyday life and in other school subjects so that they can better understand them and appreciate their usefulness.

As with everything they study in elementary school, students will find it that much easier and more rewarding to learn about mathematical reasoning and to become familiar with the required mathematical concepts and processes if learning situations are made concrete or accessible.

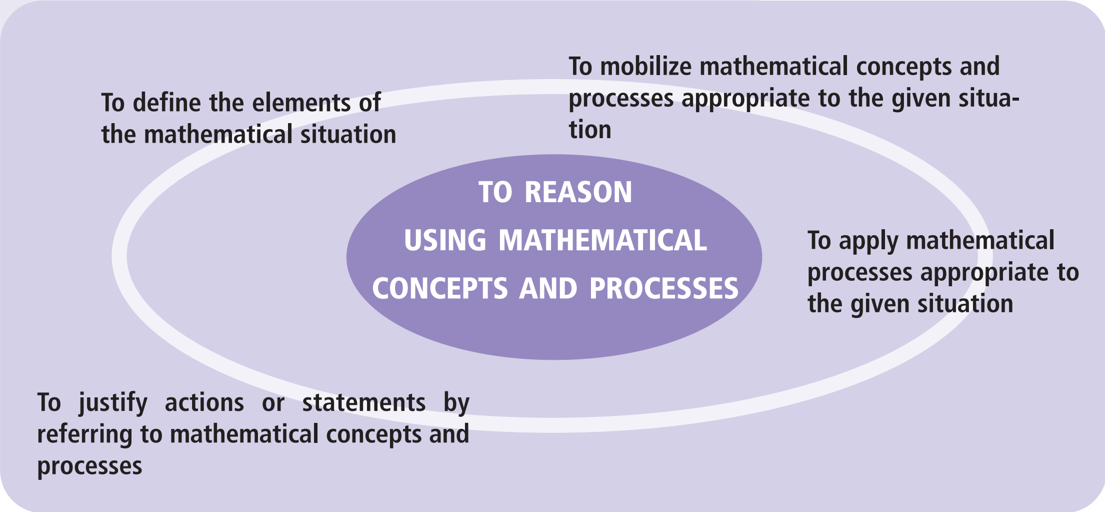
Developmental Profile

During Cycle One, the students work at putting together a network of mathematical concepts and processes. They study a few numerical patterns. They make connections between numbers and between operations and numbers. They identify easily observable geometric patterns and develop measurement sense so that they can describe and visualize their environment and move about in it. They engage in simple activities involving chance and interpret and construct graphs representing various aspects of their everyday life. They recognize situations in their immediate environment that illustrate applications of mathematics and the usefulness of technology. They also make connections between certain aspects of the history of mathematics and some of the concepts learned in class. Discussions with classmates as well as the use of technology help the students explore and develop mathematical concepts and processes.

During Cycle Two, the students develop their understanding of the number system. They describe and classify geometric objects according to their attributes. They construct more complex geometric relations and work with unconventional instruments and units of measure in studying surface areas and volumes. They continue exploring statistics and probability. By studying the history of mathematics, they make connections between the needs of societies and the development of mathematics or technology. They become more familiar with mathematical terminology, symbolism, concepts and processes.

During Cycle Three, the students develop a greater understanding of the meaning of numbers and operations. They continue studying the attributes of geometric objects, constructing geometric relationships, exploring activities involving chance and interpreting statistical data. The students identify situations in which mathematics can help them exercise critical judgment. They determine whether it is appropriate to use technology in a given activity. They continue studying the links between the various needs of modern societies and certain mathematical discoveries. They consolidate their understanding of mathematical concepts and processes.

Key Features of the Competency



Evaluation Criteria

- Appropriate analysis of a situation involving applications ❶ ❷ ❸
- Choice of mathematical concepts and processes appropriate to the given situation involving applications ❶ ❷ ❸
- Appropriate application of the chosen processes ❶ ❷ ❸
- Correct justification of actions or statements by referring to mathematical concepts and processes ❶ ❷ ❸

End-of-Cycle Outcomes

Cycle One

By the end of this cycle, the students devise and apply their own processes to do mental and written computations that involve adding and subtracting natural numbers. They construct plane figures and solids and measure lengths and time using appropriate instruments and technology.

Cycle Two

By the end of this cycle, the students continue developing and applying their own computational processes, but this time they use the four operations. They become familiar with conventional processes for written computations that involve adding and subtracting natural numbers and decimals. They can describe plane figures and solids. They begin to estimate, measure or calculate lengths, surface areas and time. They can produce frieze patterns and tessellations by means of reflections. They can do simulations related to activities involving chance and interpret and draw broken-line graphs. Without really being able to explain why, they can recognize situations in which it is appropriate to use technology.

Cycle Three

By the end of this cycle, the students mobilize their own processes as well as conventional processes to do mental and written computations involving the four operations with natural numbers and decimals. Using objects and diagrams, they start to add and subtract fractions and to multiply fractions by natural numbers. They can describe and classify plane figures, recognize the nets for convex polyhedrons, estimate, measure or calculate lengths, surface areas, volumes, angles, capacities, masses, time and temperature. They can produce frieze patterns and tessellations by means of reflections and translations, compare the possible outcomes of a random experiment with the known theoretical probabilities, calculate the arithmetic mean and interpret circle graphs. They know how to justify their use of technology.

Competency 3 • To communicate by using mathematical language

Focus of the Competency

Meaning of the Competency

The communication process benefits all those who engage in any exchange of ideas, if only because the circulation of information is mutually rewarding. It is especially useful to the person conveying a message because explaining our understanding of a situation or a concept often helps us improve or deepen that understanding.

In the specific case of mathematics, we not only derive the usual benefits associated with the communication process, but also become familiar with mathematical language. The students interpret or produce a message (oral or written or in the form of a drawing) involving a line of questioning, an explanation or a statement related to mathematical activities dealing with arithmetic, geometry, measurement, statistics and probability.

When communicating by using mathematical language, the students learn to name the processes and concepts they have studied in various activities, and this enables them to reinforce their understanding of these concepts and processes. They observe how this language can be used to understand other subjects and everyday activities. They sometimes study the development of this language throughout history.

Connections to Cross-Curricular Competencies

When students pay attention to the accuracy and clarity of their mathematical message, to the medium used to present their message and to the people to whom this message is addressed, they develop certain cross-curricular competencies, in particular the ones relating to the ability to communicate and to use information.

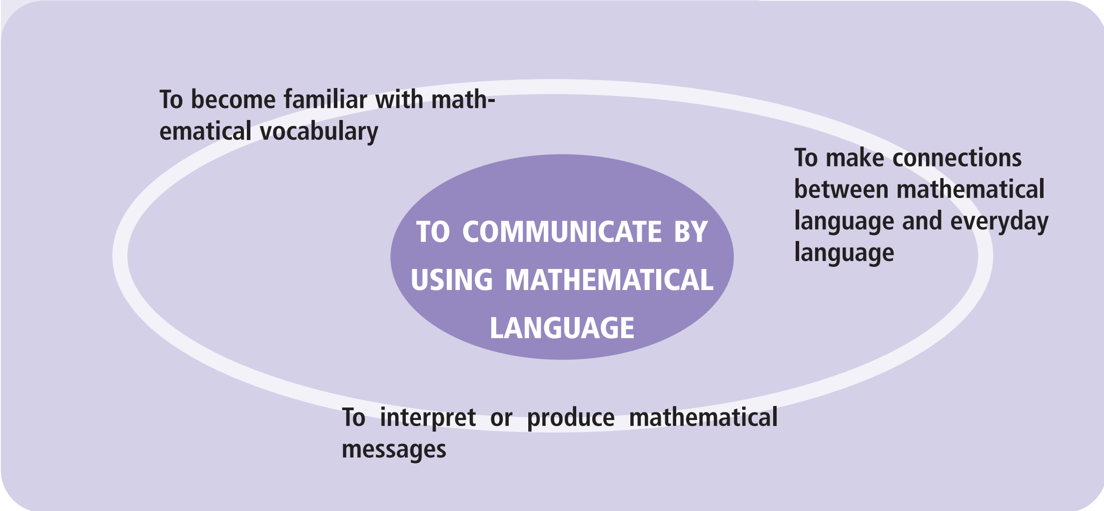
Context for Learning

Students must communicate at different stages in the learning process: when becoming familiar with a situational problem they are asked to solve and when presenting possible solutions, comparing their points of view or explaining their results. There are many examples of this type of communication: in arithmetic, for example, the students may be asked to formulate a situational problem that their classmates will have to solve. In geometry, they will make a scale drawing of a house that someone else will have to build.

Developmental Profile

During Cycle One, the students become familiar with the meaning of certain mathematical terms and symbols and learn how to use them to express their ideas and comment on those of others. During Cycle Two, they continue learning about mathematical language by making a greater effort to distinguish between the meanings of terms and symbols and by referring to different information sources. They take part in discussions with their classmates and compose simple messages. During Cycle Three, they refine their choice of the mathematical terms and symbols they use to communicate information and can give more precise explanations of their different meanings. They compare information from various sources. In discussions with classmates, they make connections between other points of view and their own opinions and adjust their message, if necessary.

Key Features of the Competency



Evaluation Criteria

- Correct interpretation of a message (oral or written) using mathematical language ❶ ❷ ❸
- Correct production of a message (oral or written) using mathematical language ❶ ❷ ❸

End-of-Cycle Outcomes

Cycle One

By the end of this cycle, the students interpret or produce a message (oral or written) such as a statement, process, or solution by using simple mathematical language and at least one of the following types of representations: objects, drawings, tables, graphs, symbols or words.

Cycle Two

By the end of this cycle, the students interpret or produce a message (oral or written) by using elaborate mathematical language and more than one type of representation, including diagrams.

Cycle Three

By the end of this cycle, the students interpret or produce a message (oral or written) by using exact mathematical language and several types of representations.

Essential Knowledges

Although science and technology are not part of the Cycle One program, many basic learnings must be covered during this cycle in the other subjects. The study of mathematics provides an ideal opportunity to acquire these learnings because of its connections with science and technology.

As with the rest of the essential knowledges, the study of measurement helps the students acquire the competency developed in Cycle One science and technology. As part of an introduction to the international system of measurement, measurements can, for example, be used in collecting information during scientific-style experiments, in making a simple technological object such as scale drawing of the classroom or a lever, or in making cutouts.

Because the mathematical concepts covered at the elementary level are connected to real-world situations, there are many opportunities to examine the mathematical, scientific and technological aspects of a learning situation simultaneously.

Learning and Strategy

Arithmetic: understanding and writing numbers

- **Natural numbers**

– natural numbers less than 1000 (units, tens, hundreds): reading, writing, digit, number, counting, one-to-one correspondence, representation, comparison, classification, order, equivalent expressions, writing numbers in expanded form, patterns, properties (even numbers, odd numbers), number line ❶

– natural numbers less than 100 000 (thousands, ten thousands): reading, writing, representation, comparison, classification, order, equivalent expressions, writing numbers in expanded form, patterns, properties (squares, prime and compound numbers), number line ❷

– natural numbers less than 1 000 000 (hundred thousands): reading, writing, representation, comparison, classification, order, equivalent expressions, writing numbers in expanded form, patterns, number line ❸

– power, exponent ❸

– approximation ❶ ❷ ❸

- **Fractions**

– fractions related to the student's everyday life ❶

– fractions based on a whole or a collection of objects: reading, writing, numerator, denominator, various representations (using objects or pictures), equivalent parts, comparison with 0, $\frac{1}{2}$ and 1 ❷

– fractions: reading, writing, numerator, denominator, various representations, order, comparison, equivalent expressions, equivalent fractions ❸

– percentages ❸

- **Decimals**

– up to two decimal places (tenths, hundredths): reading, writing, various representations, order, equivalent expressions, writing numbers in expanded form ❷

– up to three decimal places (tenths, hundredths, thousandths): reading, writing, various representations, order, equivalent expressions, writing numbers in expanded form ❸

– approximation ❷ ❸

- **Using numbers**

– converting from one type of notation to another: writing fractions, decimal numbers or percentages ❸

– choosing the most suitable notation for a given context ❸

Arithmetic: understanding and writing numbers (cont.)

• Integers

	Cycle 1	Cycle 2	Cycle 3
– reading, writing, comparison, order, representation			❸

Arithmetic: meaning of operations involving numbers

• Natural numbers

	Cycle 1	Cycle 2	Cycle 3
– operation, operation sense: addition (adding, uniting, comparing), sum, subtraction (taking away, complement, comparing), difference, term, missing term, number line, multiplication (repeated addition, Cartesian product) and division (repeated subtraction, sharing, number of times *x* goes into *y*)	❶		
– choice of operation: addition, subtraction	❶		
– operation sense: multiplication (e.g. repeated addition, Cartesian product), product, factor, multiples of a natural number, division (repeated subtraction, sharing, number of times *x* goes into *y*), quotient, remainder, dividend, divisor, set of divisors of a natural number, properties of divisibility		❷	❸
– choice of operation: multiplication, division		❷	❸
– meaning of an equality relation (equation), meaning of an equivalence relation	❶	❷	❸
– relationships between the operations	❶	❷	❸
– property of operations: commutative law	❶		
– property of operations: associative law		❷	
– property of operations: distributive law			❸
– order of operations (series of operations involving natural numbers)			❸

• Decimals

	Cycle 1	Cycle 2	Cycle 3
– operation sense: addition and subtraction		❷	
– operation sense: multiplication and division			❸

• Fractions

	Cycle 1	Cycle 2	Cycle 3
– operation sense (using objects and diagrams): addition, subtraction and multiplication by a natural number			❸

Arithmetic: operations involving numbers

• Natural numbers

	Cycle 1	Cycle 2	Cycle 3
– approximating the result of an operation: addition, subtraction	❶		
– approximating the result of an operation: addition, subtraction, multiplication, division		❷	❸
– own processes for mental computation: addition, subtraction	❶		
– own processes for mental computation: addition, subtraction, multiplication, division		❷	❸
– operations to be memorized:			
- additions ($0 + 0$ to $10 + 10$) related to the corresponding subtractions	❶		
- multiplications (0×0 to 10×10) related to the corresponding divisions		❷	
– own processes for written computation: addition, subtraction	❶		
– own processes for written computation: multiplying a three-digit number by a one-digit number		❷	
– own processes for written computation: dividing a three-digit number by a one-digit number		❷	
– conventional processes for written computation: adding two four-digit numbers		❷	
– conventional processes for written computation: subtracting a four-digit number from a four-digit number such that the difference is greater than 0		❷	
– conventional processes for written computation: multiplying a three-digit number by a two-digit number			❸

Arithmetic: operations involving numbers (cont.)

– conventional processes for written computation: dividing a four-digit number by a two-digit number, expressing the remainder as a decimal that does not go beyond the second decimal place			3
– series of operations in accordance with the order of operations			3
– patterns: series of numbers, family of operations	1	2	3
– finding prime factors		2	3
– divisibility by 2, 3, 4, 5, 6, 8, 9, 10			3

• Decimals

– approximating the result of an operation		2	3
– mental computation: addition, subtraction		2	
– mental computation: addition, subtraction, multiplication, division			3
– written computation: addition, subtraction; the result must not go beyond the second decimal place		2	
– written computation: multiplication whose product does not go beyond the second decimal place, division by a natural number less than 11			3
– mental computation: multiplication and division of decimals by 10, 100, 1000			3

• Fractions

– establishing equivalent fractions			3
– reducing fractions, irreducible fractions			3
– adding fractions using objects and diagrams, when the denominator of one fraction is a multiple of the denominator of the other fraction			3
– subtracting fractions using objects and diagrams, when the denominator of one fraction is a multiple of the denominator of the other fraction			3
– multiplying a natural number by a fraction, using objects and diagrams			3

Geometry: geometric figures and spatial sense

• Space

– locating objects and getting one's bearings in space, spatial relationships (e.g. in front, on, to the left)	1		
– locating objects on an axis	1	2	3
– locating objects in a plane	1	2	
– locating objects in a Cartesian plane		2	3

• Solids

– comparing and constructing prisms, pyramids, spheres, cylinders, cones	1		
– comparing objects in the environment with solids	1		
– attributes (number of faces, base): prisms, pyramids	1		
– describing prisms and pyramids in terms of faces, vertices and edges		2	
– nets for prisms and pyramids		2	
– classification of prisms and pyramids		2	
– recognizing nets for convex polyhedrons			3
– testing Euler's theorem (relationship between faces, vertices and edges of a convex polyhedron)			3

• Plane figures

– comparing and constructing figures made with closed curved lines or closed straight lines	1		
– identifying a square, rectangle, triangle, circle and rhombus	1		
– describing a square, rectangle, triangle and rhombus	1		
– describing convex and nonconvex polygons		2	
– describing quadrilaterals, including trapezoids and parallelograms: parallel segments, perpendicular segments, right angles, acute angles, obtuse angles		2	

Geometry: geometric figures and spatial sense (cont.)

– classifying quadrilaterals		❷	
– constructing parallel lines and perpendicular lines		❷	
– describing triangles: right triangles, isosceles triangles, scalene triangles, equilateral triangles			❸
– classifying triangles			❸
– measuring angles in degrees using a protractor			❸
– studying the features of a circle: radius, diameter, circumference, central angle			❸

• **Frieze patterns and tessellations**			
– observing and producing patterns using geometric figures		❷	
– congruent figures	❶		
– observing and producing (grids, tracing paper) frieze patterns by means of reflections: reflection, line of reflection		❷	
– observing and producing tessellations by means of reflections		❷	
– observing and producing (grids, tracing paper) frieze patterns by means of translations: translation, translation arrow (length, direction, sense)			❸
– observing and producing tessellations by means of translations			❸

Measurement

• **Lengths: estimating and measuring**			
– dimensions of an object	❶		
– unconventional units: comparison, construction of rulers	❶		
– conventional units (m, dm, cm)	❶		
– conventional units (m, dm, cm, mm)		❷	
– conventional units (km, m, dm, cm, mm)			❸
– relationships between units of measure		❷	❸
– perimeter, calculating the perimeter		❷	

• **Angles: estimating and measuring**			
– comparing angles (right, acute, obtuse)		❷	
– degree			❸

• **Surface areas: estimating and measuring**			
– unconventional units		❷	
– conventional units (m^2, dm^2, cm^2), relationships between the units of measure			❸

• **Volumes: estimating and measuring**			
– unconventional units		❷	
– conventional units (m^3, dm^3, cm^3), relationships between the units of measure			❸

• **Capacities: estimating and measuring**			
– unconventional units			❸
– conventional units (L, mL), relationships between the units of measure			❸

• **Masses: estimating and measuring**			
– unconventional units			❸
– conventional units (kg, g), relationships between the units of measure			❸

• **Time: estimating and measuring**			
– conventional units, duration (day, hour, minute, second, daily cycle, weekly cycle, yearly cycle)	❶	❷	
– relationships between the units of measure			❸

• **Temperatures: estimating and measuring**			
– conventional units (°C)			❸

Statistics

• Formulating questions for a survey	1	2	3
• Collecting, describing and organizing data using tables	1	2	3
• Interpreting data using a bar graph, a pictograph and a data table	1		
• Displaying data using a bar graph, a pictograph and a data table	1		
• Interpreting data using a broken-line graph		2	
• Displaying data using a broken-line graph		2	
• Interpreting data using a circle graph			3
• Arithmetic mean (meaning, calculation)			3

Probability

• Experimentation with activities involving chance	1	2	3
• Predicting the likelihood of an event (certainty, possibility or impossibility)	1	2	3
• Enumerating the possible outcomes of a simple random experiment	1		
• Probability that a simple event will occur (more likely, just as likely, less likely)		2	3
• Enumerating the possible outcomes of a random experiment using a table, a tree diagram		2	3
• Comparing the outcomes of a random experiment with known theoretical probabilities			3
• Doing simulations with or without a computer		2	3

Cultural References

• Numbers			
– origin and creation of numbers	1		
– development of systems for writing numbers	1		
– number systems (*e.g. Arabic, Roman, Babylonian, Mayan*): characteristics, advantages and disadvantages		2	3
– social context (*e.g. price, date, telephone, address, age, quantity: mass, size*)	1	2	3
• Operations			
– own or conventional computation processes: development, limitations, advantages and disadvantages	1	2	3
– technology: development (*e.g. sticks, strokes, abacus, calculator, software*), limitations, advantages and disadvantages	1	2	3
– symbols (origin, development, need, mathematicians involved): $+, -, >, <, =$	1		
– symbols (origin, development, need, mathematicians involved): $\infty, \div, \neq$		2	
– interdisciplinary or social context (*e.g. history, geography, science and technology*)	1	2	3
Geometric figures			
– interdisciplinary or social context (e.g. architecture, maps, arts, decoration)	1	2	3
– symbols (origin, development, need, mathematicians involved): $\angle$, //, $\perp$		2	3

• **Measurement**			
– systems of measurement (historical aspect)	❶	❷	❸
– units of measure: development according to society's needs (*e.g. agrarian measurements, astronomy, standard measurement, precision*), instruments (rudimentary approach for measuring time, hourglass, clock)		❷	❸
– symbols (origin, development, need): m, dm, cm	❶		
– symbols (origin, development, need): m, dm, cm, mm		❷	
– symbols (origin, development, need): km, m, dm, cm, mm			❸
– symbols (origin, development, need): kg, g, L, mL			❸
– symbols (origin, development, need): h, min, s	❶	❷	❸
– symbols (origin, development, need): °C			❸
– symbols (origin, development, need, mathematicians involved): ( ), %			❸

In each cycle, students in a given class carry out at least one individual or group project or activity related to cultural references.

Symbols

• 0 to 9, +, –, >, <, =	❶		
• 0 to 9, +, –, ×, ÷, >, <, =, ≠		❷	
• 0 to 9, +, –, ×, ÷, >, <, =, ≠, ( ), %			❸
• Calculator keys [keys: 0 to 9, +, –, ×, ÷, =, ON, OFF, AC, C, CE (all clear, clear, clear last entry)]	❶	❷	❸
• Certain commonly used calculator functions [memories (M+, M–, MR, MC), change of sign (+/–)]			❸
• Numbers written using digits	❶	❷	❸
• Writing fractions ($\frac{a}{b}$)	❶	❷	❸
• Writing decimals using a period as the decimal marker		❷	❸
• Exponential notation $■^2$, $■^3$			❸
• ∠, //, ⊥		❷	❸
• m, dm, cm	❶		
• m, dm, cm, mm		❷	
• km, m, dm, cm, mm			❸
• kg, g, L, mL			❸
• h, min, s (representation of time of day: 02:00, 2:00 a.m; representation of elapsed time: 2 h 10 min, 2:10)	❶	❷	❸
• °C			❸
• $, ¢		❷	❸

Vocabulary

❶

addition
as many as
as much as

bar graph
base of a solid

centimetre
certain event
chance
circle
cone
cube
curved line
cylinder

day
decimetre
decreasing order
depth
difference
digit

even number

face
fewer
fraction

grouping

half
height
hour
hundreds place

impossible event
increasing order
…is bigger than…
…is equal to…
…is smaller than…

length
less

metre
minus
minute
more

natural number
none
number
number line

odd number
one third
one-to-one correspondence

pictograph
plane figure
plus
possible event
prism
probable outcome
pyramid

quarter

rectangle
rhombus

Vocabulary

1

second
series
side
solid
sphere
square
straight line
subtraction
sum
survey

table
tens place
triangle

unit
unit of measure

width

2

acute angle
angle
area
at least
at most

base ten
broken-line graph

Cartesian plane
chance (statistics)
compound number
convex polygon
curved body
curved surface

daily cycle
decimal
denominator
dividend
dividing up
division
divisor

edge
equality
equation
equivalent part
event

factor
flat surface
frieze pattern

gram

hundredth

inequality
inverse operation
parallelogram
…is greater than…
…is less than…
…is not equal to…
…is parallel to…
…is perpendicular to…

just as likely

kilogram

less likely
line of reflection

measuring instrument
millimetre
missing term
more likely
multiple
multiplication

net of a solid
nonconvex polygon
number squared
numerator

obtuse angle
ordered pair

perimeter
place value
plane
polygon
prime number
product

quadrilateral
quotient

reference system
reflection
remainder
right angle

segment
surface
symmetric figure

ten thousands place
tenth
term
tessellation
thousand
thousands place
trapezoid
tree diagram

vertex
volume

weekly cycle
whole

yearly cycle

3

arithmetic mean

capacity
central angle
circle graph
circumference
convex polyhedron
cube of (the)

degree (angle)
degree Celsius
diameter

equilateral triangle
equivalent fraction
Euler’s theorem
exponent

hundred thousands place

integer
irreducible fraction
isosceles triangle

kilometre

litre

mass
millilitre
million

negative number

parenthesis

percentage
polyhedron
positive number
power
protractor

radius
right triangle

scalene triangle
square of (the)

thousandth
translation
translation arrow

Suggestions for Using Information and Communications Technologies*

Activity	Cycle 1	Cycle 2	Cycle 3
• Becoming familiar with the basic operations of a calculator [keys: 0 to 9, +, –, ×, ÷, =, ON, OFF, AC, C, CE (all clear, clear, clear last entry), functions: recursive with the = key]	❶	❷	❸
• Becoming familiar with certain commonly used calculator functions [memories (M+, M–, MR, MC), change of sign (+/–)]			❸
• Using technology for operations involving numbers that go beyond the scope of the material covered in these cycles	❶	❷	❸
• Using technology to present proofs related to operations	❶	❷	❸
• Using a calculator in applying different problem-solving strategies	❶	❷	❸
• Using a calculator and a computer to explore natural numbers and operations	❶	❷	❸
• Using a calculator and a computer to explore decimals, fractions and operations		❷	❸
• Using a calculator and a computer to explore integers			❸
• Using a computer (graphics and spreadsheet software as well as simulations) in applying different problem-solving strategies	❶	❷	❸
• Using a computer (word-processing, graphics and spreadsheet software) to present information related to the solution	❶	❷	❸
• Producing a drawing (solids, plane figures, frieze patterns and tessellations) using graphics software	❶	❷	❸
• Using a computer to look for information		❷	❸
• Learning to collect data using spreadsheet software		❷	❸
• Learning to produce a graphic representation of data using spreadsheet software		❷	❸
• Learning to do a computer simulation of a random experiment		❷	❸
• Using the Internet to find historical accounts related to concepts studied in class		❷	❸
• Consulting Internet Web sites on mathematics as well as glossaries and databases		❷	❸
• Using interactive mathematics sites			❸

* The use of information and communications technologies is compulsory, but it is up to the teacher to choose the activities involving ICT.